PHY680: Advanced Nonperturbative QFT

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YITP seminar room (Math 6-125), Tu-Th 11:30am–12:50pm, Fall '22

In this class we will cover a circle of ideas and techniques in quantum field theory (QFT), organized around the fundamental question: *Given a concrete QFT model, what is its long-distance behavior?* The concrete QFT may have a standard presentation in terms of a Lagrangian, or may be given more abstractly in terms of a conformal field theory (CFT) perturbed by a relevant operator. Does the theory have a mass gap? If so, is the long-distance physics completely trivial or is governed by a non-trivial topological QFT (TQFT)? If the theory does *not* have a mass gap, does it flow in the IR to an interacting CFT or to a free CFT? Are the global symmetries (continuous or discrete) of the theory preserved by the vacuum or spontaneously broken? In general these are difficult questions to answer, but an experienced researcher can often come up with an educated guess, or at least enumerate some plausible options. The ambitious aim of this class is to teach you some of the qualitative ideas and quantitative tools that will make you such an experienced researcher. We will cover both very old and very new ideas, and illustrate them in a plethora of examples in various spacetime dimensions – mostly $D = 4$ and $D = 3$, but with excursions to $D = 2$ and $D > 4$. Some of the ideas and tools that we'll try to cover:

(a) Review of the renormalization group (RG) and of effective field theory (EFT) ideas. Topology of the RG.

(b) Review of basic ideas about CFT. Conformal perturbation theory.

(c) c-theorems in various dimensions.

(d) Modern theory of symmetries. Ordinary and generalized symmetries (including higher-form and non-invertible symmetries). 't Hooft anomalies and their matching. Symmetry protected topological phases in condensed matter.

(e) Exact dualities and IR dualities.

Some of the concrete examples:

1. Scalar and Yukawa theories in $D < 4$.
2. Particle/vortex duality in $D = 3$. Web of $D = 3$ dualities involving Chern-Simons-matter theories.
3. Non-abelian gauge theories in $D = 4$. Confinement, chiral symmetry breaking, chiral Lagrangians. Phases of gauge theories. Conformal window of QCD.
4. Time permitting, supersymmetric examples with 4 supercharges.
   In $D = 4$, $\mathcal{N} = 1$ super QCD and Seiberg duality. In $D = 3$, $\mathcal{N} = 2$ mirror symmetry.
5. (Unlikely we'll get this far but we'll see:) Supersymmetric examples with 8 supercharges.
   In $D = 4$, Seiberg-Witten theory. In $D = 3$, $\mathcal{N} = 4$ mirror symmetry.

A notable omission will be a more systematic treatment of $D = 2$. This would require advanced knowledge of 2D CFT (chiral algebras, rational CFTs) that seems too much to take for granted.

Prerequisites. Everyone is welcome, but it's unlikely you will get much out of this class unless you have already taken a full year course in QFT, or have mastered that material by yourself (at the level say of Peskin-Schroeder or Srednicki). I will assume that you are comfortable with writing Lagrangians with scalars, fermions and gauge fields and the basic ideas about renormalization (e.g. that you can compute a simple one-loop Feynman integral and know what β functions and anomalous dimensions are). On the other hand, the emphasis will be very different from traditional QFT classes and you may benefit from being exposed to certain qualitative ideas that require little technical knowledge. More than any concrete technical prerequisites I think a certain maturity may be needed, as we will connect a large set of ideas ranging from high-energy physics to statistical mechanics, and jump between disparate examples in various spacetime dimensions. If we get to supersymmetric examples, I will try to give a self-contained intro to the basics ideas and tools of supersymmetry, but of course some previous exposure wouldn't hurt.