

# AMS Foundation Exam (Jan. 2024) Probability Questions

Date of Exam: Jan. 18, 2024  
Time of Exam: 11:15am–1:15pm

There are 3 problems, are you are required to solve all of them. All problems are weighted equally. Please show detailed work for full credit.

Academic integrity is expected of all students at all times, whether in the presence of absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination.

NAME: \_\_\_\_\_ ID: \_\_\_\_\_

Signature: \_\_\_\_\_

1. Let  $X$  and  $Y$  be discrete random variables with joint probability mass function

$$f(x, y) = \frac{C}{(x + y - 1)(x + y)(x + y + 1)}, \quad x, y = 1, 2, 3, \dots$$

Find the constant  $C$  and the marginal mass function of  $X$ .

2. A factory has produced  $n$  machines, each of which is defective with probability  $p$ . For each machine, a test is performed. Suppose a machine is defective, the test will detect the defect with probability  $\delta$ . Let  $X$  be the number of defective machines and  $Y$  be the number of machines detected as defective. Compute  $E[X|Y]$ .

3. Let  $X_{(1)} \leq X_{(2)} \leq \dots \leq X_{(n)}$  be the order statistics of  $X_1, X_2, \dots, X_n$ , where  $X_i$ 's are i.i.d. uniform random variables between 0 and 1. Find the probability density function of  $X_{(n)} - X_{(1)}$ .

Extra Work.