

AMS Foundation Exam (Jan. 2025) Probability Questions

Date of Exam: Jan. 23, 2025
Time of Exam: 11:15am–1:15pm

There are 3 problems, and you are required to solve all of them. All problems are weighted equally. Please show detailed work for full credit.

Academic integrity is expected of all students at all times, whether in the presence of absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination.

NAME: _____ ID: _____

Signature: _____

1. Let X_1, X_2, \dots, X_n be iid continuous random variables, where $n \geq 2$ is an integer. Find

$$P(X_1 = \min\{X_1, X_2, \dots, X_n\}, X_n = \max\{X_1, X_2, \dots, X_n\}).$$

2. Let X and Y be two independent standard normal random variables. Find the cumulative distribution function of the random variable $Z = X/Y$.

3. Let X_1, X_2, \dots be iid random variables with $E[X_n] = 0$ and $Var(X_i) = \sigma^2 < \infty$. It is known that $\lim_{n \rightarrow \infty} P(S_n > \sqrt{n}) = \frac{1}{3}$, where $S_n = \sum_{i=1}^n X_i$. Compute σ .

Extra Work.